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Spatial and Temporal Model for Electric Vehicle Rapid Charging Demand

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***Abstract*—In this paper, we present a spatial and temporal model of electric vehicle (EV) charging demand for rapid charging stations located in the urban area. For this problem three lines of investigations have been pursued in the literature: the first one is based on an assumption of a fixed charging station and charging time during the off-peak hours for EV charging demand; the second scenario has limited charging stations at typical locations instead of a mathematical model; and the third one is for charging demands in one way system, such as charging stations nearby highway exists. Hence, from the perspective of a distributed urban transportation system, EV charging demand still has unidentified quantity which varies with respect to charging station location and charging time. In this paper we presents mathematical models for EV charging demand at rapid charging stations. The rapid charging station is formulated as a $M/M/s$ queuing service system. A traffic flow based model generates the arrival rate for EVs to arrive the charging station. The computational results demonstrate that the proposed model is able to estimate the dynamic spatio-temporal charging demands in the large-scale urban traffic environment.**

***Index Terms*—electric vehicle, charging demand, energy consumption, energy management, queuing model**

I. Introduction

EVs are increasingly being seen as a sustainable type of transportation by countries worldwide which are more energy efficient than traditional vehicles and can reduce gasoline consumption and carbon emissions (see [1] etc). Several automakers, such as Nissan, Chevrolet, and Ford have manufactured commercial plug-in electric vehicles (PEVs) which appeared in market in 2010 and 2011 (see [2]). More and more traditional automakers are joining the design and manufacture of PEVs. Federal government of several countries, such as Singaporae and the U.S., have developed EV test-beds which can be used to analyze benefits and feasibility of EVs' adoption for urban transportation and power grid system. Kintner-Meyer et al. [3] concluded that the transportation electrification can fuel up to 84% of the U.S.'s light-duty vehicle fleet with existing electricity infrastructure. However, they also demonstrated that high plug-in hybrid electric vehicle (PHEV) and PEV penetration may lead to negative impact on extreme events or cause grid emergencies to existing power gird, which brings higher electricity cost, lower reliablity, and other side effects. Hence, the spatio-temporal charging demands of EVs are fundamental questions for EV adoption in urban traffic environment and highway systems. The accurate estimation of EV charging demands would help determine the power supply of existing charging station, new charging station placement, vehicle to grid (V2G) implementation, and the reliability of existing power grid. This paper presents a spatio-temporal model of EVs' charging demand for rapid charging stations in urban environment.

Although the spatio-temporal EV charging demand is important for EV adoption in urban or highway traffic system, there are very few literature and research works on EV charging demand at rapid charging stations. We summarize these earlier works into three categories. The first one has the assumption that charging demands are fulfilled at a fixed charging station with fixed charging time (see [3] etc), for instance, at a resident slow charging station in the evening or at night. But the operation of special purpose or commercial PEVs, such as taxis and police cars, requires them to be charged anytime at rapid charging stations instead of a single residential charging station with specified time. Second, the survey on EV charging demands at various charging locations has been studied in [4] etc. They anticipated EV charging demands at residential, office, and shopping areas based on the survey from potential EV users. However, EVs are newly adopted by most cities hence there is no historical data to help EV surveys and EV potential user estimation and no mathematical models have been developed. Third, Bae and Kwasinski [5] studied the spatio-temporal model of EVs on a one-way high way system using a fluid dynamic traffic model and the $M/M/s$ queuing system. They considered traffic model for one direction on the highway and obtained the charging EV behavior by an assumed split-ratio for departure number of vehicles and charging vehicles at a highway exist.

In this paper, to the best of our knowledge, our contributions lay in the spatio-temporal charging model for EV charging demand with multiple charging stations in complicated urban traffic environment, instead of charging demand in a single charging station or in the one way traffic system.

The rest of the paper is organized as follows. In Section II, we provide the problem description and the corresponding formulation. In Section III, we present the computational results of our model in Singapore traffic network with fast charging facilities. In Section IV, we provide a conclusion

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and future research direction for this paper.

II. Problem Description and Formulation

In this section, we provide the formal problem description and demonstrate mathematical model for spatio-temporal charging demand of rapid charging stations. We let $G(N, A)$ denote the traffic network with node set N and arc set A. We consider a model which integrates cell transmission model and queuing model for transportation system and rapid charging facility service, respectively. The cell transmission based transportation models are discussed by Daganzo in [6] and [7] and Ziliaskopoulos in [8]. The cell transmission model is a hydrodynamic model utilizing differential equations, where the difference equations are simplified with the assumption that a piecewise linear relationship between flow and density in the cell level. It assumed that a cell of a segment on a traffic link whose length is equal to the traveling distance of free-flow vehicle during a fixed time interval τ.

Based on the cell concept, Ziliaskopoulos in [8] categorized the set of cells into five groups: (1) source cell, (2) sink cell, (3) diverging cell, (4) merging cell, and (5) ordinary cell and illustrated them in the following way.

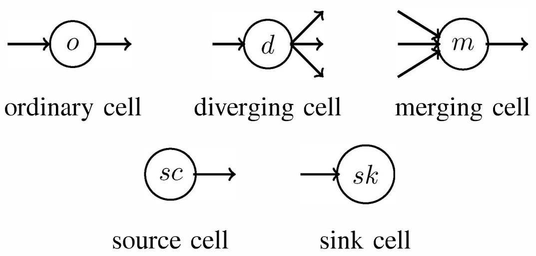

Fig. 1. Categorization of cells in cell transmission model

A. Cell Transmission Model for EV Traffic Flow

Given the location of a rapid charging facility, we assume that the traffic flow on a cell i at time t can then be splitted into two types, vehicles driving on the traffic link and vehicles staying on a charging station. After charging, the vehicles would be merged back to the traffic flow. Hence, we create two cell for each rapid charging facilities. These cells are connected with their upperstream and downstream cells. We assume that the split-ratio of traffic flow is known for regular driving and charging behavior and denote them as α_1 and α_2 with $\alpha_1 + \alpha_2 = 1$. In addition, we define the ***charging cell*** as a special set of the ordinary cells with notation $\mathcal{C}_c$ and all successor and predecessor neighboring cells of charging cells are diverging and merging cells, respectively.

We let $\mathcal{I}$ be the whole set of cells in the traffic network G and $\mathcal{T}$ be the whole time horizon. We consider the discredited time interval, $t = 1, 2, \ldots, |\mathcal{T}| = T$. Given a cell i, we let $\delta^+(i)$ be the set of successor cells of cell i and $\delta^-(i)$ be the set of predecessor cells of cell i. We let $\mathcal{C}_{ord}$, $\mathcal{C}_{divg}$, $\mathcal{C}_{mrg}$, $\mathcal{C}_{sc}$, $\mathcal{C}_{sk}$, $\mathcal{C}_{cg}$, $\mathcal{C}_c$ be cell sets for ordinary cells, diverging cells, merging cells, source cells, sink cells, and charging facility

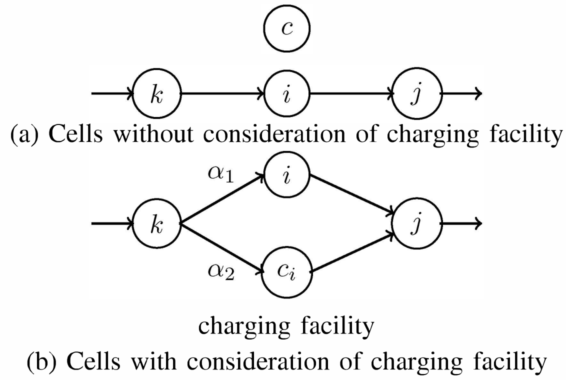

Fig. 2. Categorization of cells in cell transmission model

cells, respectively. Note here that every cell $i \in \mathcal{C}_c$ would be splitted into two new cells i_1 and i_2 which represent the cell with regular traffic flow (bypassing the charging station) and the one with traffic entering entering a charging station, respectively.

Next, we introduce variables utilized in the cell transmission based EV traffic flow model.

- x_i^t : the number of vehicles contained in each cell i in time interval t
- N_i^t : the maximum number of vehicles in cell i in time interval t
- Q_i^t : the maximum number of vehicles that can flow in or out from cell i during time interval t
- y_{ij}^t : the number of vehicle moving from cell i to j during time interval t
- d_i^t : supply (inflow) at cell i in time interval t
- $\widetilde{d}_i^t$: demand (outflow) at cell i in time interval t

We first present the model for the cell transmission based traffic flow on source and sink cells, respectively.

$$x_i^t = x_i^{t-1} + d_i^{t-1} - y_{ij}^{t-1}, \quad i \in \mathcal{C}_{sc}, j \in \delta^-(i), t \in \mathcal{T} \quad (1)$$
$$x_i^t = x_i^{t-1} - \widetilde{d}_i^{t-1} + y_{ij}^{t-1}, \quad i \in \mathcal{C}_{sk}, j \in \delta^+(i), t \in \mathcal{T} \quad (2)$$

The initial value $x^0 + i$ can be set to the initial traffic condition of the network at the beginning of the time period of interest. Constraints (1) and (2) are the flow mass conservation constraints, which guarantee the flow coming in, going out of source and sink cells and the flow staying on cells would be balanced.

Second, we present the flow conservation constraint for ordinary, diverging, and merging cells.

$$x_i^t = x_i^{t-1} - \sum_{j \in \delta^-(i)} y_{ij}^{t-1} + \sum_{j \in \delta^+(i)} y_{ij}^{t-1}, \qquad i \in \mathcal{C}_{ord} \cup \mathcal{C}_{divg} \cup \mathcal{C}_{mrg}, t \in \mathcal{T} \quad (3)$$

Third, we present bounded flow constraints for the flow staying on, coming in, and going out of a cell.

$$\sum_{y\in\delta_t^+} y_{ij}^t \le x_i^t, \qquad i,j\in C, t\in\mathcal{T} \tag{4}$$

$$\sum_{y\in\delta_t^+} y_{ij}^t \le Q_i^t, \qquad i,j\in C, t\in\mathcal{T} \tag{5}$$

$$\sum_{y\in\delta_t^-} y_{ij}^t \le Q_j^t, \qquad i,j\in C, t\in\mathcal{T} \tag{6}$$

$$\sum_{y\in\delta_t^-} y_{ij}^t \le \frac{v}{w}(N_j^t - x_j^t), \qquad i,j\in \mathcal{C}, t\in\mathcal{T} \tag{7}$$

$$\sum_{y\in\delta_t^+} y_{ij}^t \le \frac{v}{w}(N_j^t - x_j^t), \qquad i,j\in \mathcal{C}, t\in\mathcal{T} \tag{8}$$

Here v is the free-flow speed and w is the backward propagation speed. Constraint (4) presents that the flow between any two cells is bounded by the occupancy of the beginning and end cells; constraints (5) and (6) define the remaining capacity of the beginning and ending cells; and constraints (7) and (8) represent the maximum flow which can get out of the beginning cell and get into ending cells, respectively.

The above constraints completely formulate the EV traffic flow model without considering charging facilities and their correponding ordinary cells' coming-in, going-out, and staying-on flow. The objective function considered here is to minimize the total traveling time during the whole assignment period $\mathcal{T}$ and formulated as follows:

$$\min_{x,y} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{C}\setminus\mathcal{C}_s} \tau x_i^t \tag{9}$$

Due to that τ is a constant time interval, the above objective (9) could be rewritten as

$$\min_{x,y} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{C}\setminus\mathcal{C}_s} x_i^t \tag{10}$$

Without the consideration of charging stations, Ziliaskopoulos presented in [8] the necessary and sufficient condition for the optimal condition of a system with a single destination node.

Theorem 1 (Ziliaskopoulos [8]): A necessary and sufficient condition for system optimum dynamic traffic assignment on a single destination network is that all used paths from any cell and departure time interval to the destination cell have cost equal to the marginal cost of an additional unit of demand at that cell and time interval, while all unused paths have cost higher than or equal to the marginal cost.

B. Charging Station Service Model

In this section, we present the charging station service model as an $M/M/s$ queue with the following assumptions.

- The arrival process of vehicles to a rapid charging facility can be described by a Poisson process (and thus the interarrival time are exponentially distributed) with rate λ.
- The number of charging pumps in a charging station located in cell i is $s(i)$ with $i\in\mathcal{C}(c)$.
- The charging service time for all pumps is the same and equal to μ, which is independently and exponentially distributed.
- All vehicles entering a charging station served by charging pumps follow the first-in-first-out rules (FIFO). Meanwhile, vehicles leave charging stations directly when charging is finished.

Following the cell transmission model, vehicles entering the charging station are seperated from the traffic entering cell i and have ratios $\alpha_2(i)$, $i\in\mathcal{C}_c$. Then, the following conclusion holds.

Proposition 1: Following the cell transmission traffic flow model, the arrival rate for vehicles entering charging station $\lambda(i) = \alpha_2(i)\sum_{k\in\delta^+(i)} y_{ki}^t$ with $i\in\mathcal{C}(c)$.

Based on above assumptions, the charging service in a charging station follows the $M/M/s$ queue, whose arrival rate and the departure rates depend on the number of vehicles in the system, is known as a birth and death queueing model (see [9]).

Proposition 2: The departure rates of vehicles from a charging station in cell i with $s(i)$ charging pumps can be formulated as follows.

$$\nu_{i_2}^t = \begin{cases} \mu x_{i_2}^t, & \text{if } x_{i_2}^t \le s(i), \\ \mu s(i), & \text{if } x_{i_2}^t > s(i), \end{cases} \quad i\in\mathcal{C}_c \tag{11}$$

C. The Formulation for Charging Cells

Now we discuss the model for charging cell i at time interval t, $i\in\mathcal{C}(c)$ and $t\in\mathcal{T}$

$$\sum_{j\in\delta^-(i)} y_{i_2 j}^{t-1} = \nu_{i_2}^t, \qquad i\in\mathcal{C}_c \tag{12}$$

$$x_{i_1}^t = x_{i_1}^{t-1} + \alpha_{i_1}\sum_{j\in\delta^+(i)} y_{ij}^{t-1} - \sum_{j\in\delta^-(i)} y_{i_1 j}^{t-1}, \quad i\in\mathcal{C}_c \tag{13}$$

$$x_{i_2}^t = x_{i_2}^{t-1} + \alpha_{i_2}\sum_{j\in\delta^+(i)} y_{ij}^{t-1} - \sum_{j\in\delta^-(i)} y_{i_2 j}^{t-1}, \quad i\in\mathcal{C}_c \tag{14}$$

With Condition (11) and Constraint (12), Constraint (12) could be re-written as follows:

$$\sum_{j\in\delta^-(i)} y_{i_2 j}^{t-1} \ge \mu x_{i_2}^t - M\eta_i^t \tag{15}$$

$$\sum_{j\in\delta^-(i)} y_{i_2 j}^{t-1} \ge \mu s(i) - M(\eta_i^t - 1) \tag{16}$$

$$x_{i_2}^t - s(i) \le M\eta_i^t, \qquad i\in\mathcal{C}_c \tag{17}$$

where M is a given big number. Constraint (17) determines the larger value between x_{i2}^t and $s(i)$. Then, Constraints (15) and (16) push the outflow from a charging station to be equal to the departure rate of the charging station.

D. Cell Transmission Model for EV Traffic Flow

Now, we conclude the model for the EV traffic flow model as follows:

$$\min_{x,y} \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{C}\setminus\mathcal{C}_s} x_i^t$$
$$s.t. \text{Constraints (1)-(8) and (15)-(17).} \tag{18}$$

E. Spatio-temporal Charging Demand by Cell Transmission Model and M/M/s Queue

In this section, we consider the spatio-temporal charging demands based on the flow information generated from the cell transmission traffic model and $M/M/s$ queue.

Proposition 3: The steady state of $M/M/s$ queue for a charging station is

$$\frac{\alpha_{i_2} \sum_{j\in\delta^+(i)} y_{ij}^{t-1}}{\mu S(i)} \leq 1 \tag{19}$$

If the above condition does not hold, then, the queue size of the charging station would be increased without limitation. Hence, there would be no limiting probabilities for this $M/M/s$ model.

Proposition 4: The spatio-temporal charing demand at charging station i at time t with $i \in \mathcal{C}_c$ and $t \in \mathcal{T}$ is

$$\beta B_i^t \text{ with } B_i^t = \begin{cases} x_{i_2}^t, \text{ if } x_{i_2}^t < s(i) \\ s(i), \text{ otherwise,} \end{cases} \tag{20}$$

where β is the average charging power per pump.

III. Computational Results

In this section, we report our computational report for the spatio-temporal charging demand at rapid charging facility. We explore our computational report in Singapore traffic networks.

First, the total testing time horizon considered is 1 hour with small time intervals τ, each has 30 seconds. Then, there are 120 time slots in our computation. We consider the major roads, including expressway and major arterial roads, in Singapore. We assume that the speed limits on the expressway and arterial road to be 90 km/hour and 45 km/hour, respectively. We set the upper bound for flow volume on the express way to be 150 vehicles/min (see [10]) and 50 vehicles/min for major arterial road. With a small time interval $\tau = 30$ seconds, there are total 180 cells under testing. We select 5 charging stations in Singapore traffic networks, which are located in the airport, the shopping area of the Central Business District (CBD), the business area of CBD, the developing area at the east and the north sides of Singapore. In each charging station, we assume that there are 4 charging pumps. We assume that the fixed ratio of the traffic flow entering charging station over the regular traffic flow is 3/7.

Based on the EVs' traffic information during a morning peak hour, we generate the origin/destination (O/D) matrix. The EV penetration is around 10% of regular traffic in our model. In our experiment, we fix the original and destination node pairs to be the same as the seed (peak hour) O/D matrix. The flow between O/D node pairs is randomly generated based on the O/D matrix, where the mean value of the O/D flow in the seed O/D matrix has a variance $\pm 10\%$ of the seed O/D flow.

At the 20 mins time slot during the testing time horizon, the total number of charging bumps in service and the charging demand at charging stations located in the shopping area of CBD and the airport are 6 and 258 KW, respectively. At the 40 mins time slot during the testing time horizon, the total number of charging bumps in service and the charging demand at charging stations located in the shopping area of CBD and the airport are 8 and 344 KW, respectively.

IV. Conclusion and Future Research

In this paper we presented a spatio-temporal model for commercial EV charging demand at rapid charging stations. We proposed a traffic model based on dynamic traffic assignment in the urban traffic network and formulated the charging service at each placed rapid charging station as an $M/M/s$ queue. Then, we obtained the spatio-temporal charging demand at the charging station by combining the traffic flow model with charging station service model.

References

[1] M. Duvall, E. Knipping, M. Alexander, L. Tonachel, and C. Clark, "Environmental assessment of plug-in hybrid electric vehicles-volume 1: Nationwide greenhouse gas emissions," Electric Power Research Institute, Palo Alto, CA, Tech. Rep. 1015325, 2007.

[2] L. Dickerman and J. Harrison, "A new car, a new grid," *IEEE Power & Energy Magazine*, vol. 8, pp. 55–61, 2010.

[3] M. Kintner-Meyer, K. P. Schneider, and R. G. Pratt, "Impacts assessment of plug-in hybrid vehicles on electric utilities and regional us power grids: Part 1: Technical analysis," Paciffic Northwest National Laboratory, Richland, WA, Tech. Rep. PNNL-SA-61669, 2007.

[4] J. Axsen and K. S. Kurani, "Anticipating plug-in hybrid vehicle energy impacts in california: Constructing consumer-informed recharge profiles," *Transportation Research: Part D, Transport and Environment*, vol. 15, pp. 212–219, 2010.

[5] S. Bae and A. Kwasinski, "Spatial and temporal model of electric vehicle charging demand," *IEEE Transactions on Smart Grid*, vol. 3, pp. 394–403, 2012.

[6] C. F. DAGANZO, "The cell transmission model: A simple dynamic representation of highway traffic consistent with the hydrodynamic theory," *Transportation Research Part B: Methodological*, vol. 28, pp. 269–287, 1994.

[7] ——, "The cell transmission model. ii. network traffic," *Transportation Research Part B: Methodological*, vol. 29, pp. 79–93, 1995.

[8] A. K. Ziliaskopoulos, "A linear programming model for the single destination system optimum dynamic traffic assignment problem," *Transportation Science*, vol. 34, pp. 37–49, 2000.

[9] S. M. Ross, *Introduction to Probability Models*, 10th ed. Academic Press, December 2009.

[10] P. Olszewski and L. Xie, "Modeling the effects of road pricing on traffic in singapore," *Transportation Research Part A*, vol. 39, pp. 755–772, 2005.