Unified Mathematical Programming Frameworks for Survivable Logical Topology Routing in IP-over-WDM Optical Networks

Tachun Lin, Zhili Zhou, Krishnaiyan Thulasiraman, Guoliang Xue, and Sartaj Sahni

***Abstract*—The survivable logical topology routing problem in an IP-over-WDM optical network is to map each link (*u*,*v*) in the logical topology (at the IP layer) into a lightpath between the nodes *u* and *v* in the physical topology (at the optical layer) such that failure of a physical link does not cause the logical topology to become disconnected. It is assumed that both the physical and logical topologies are at least two-edge connected. For this problem, two lines of investigation have been pursued in the literature: one pioneered by Modiano and Narula-Tam [Proc. IEEE INFOCOM, 2001, p. 348] and the other pioneered by Kurant and Thiran [Proc. Int. Conf. on Broadband Networks (BROADNETS), 2004, p. 44]. Since then there has been a great deal of research on this problem. Most of the works have not considered limitations imposed on the routings by physical capacity limits and other metrics in addition to survivability. In this paper, we first introduce two concepts: weakly survivable routing and strongly survivable routing. We then provide mathematical programming formulations for two problems. The first problem is to design a survivable lightpath routing that maximizes the logical demand satisfaction before and after a physical link failure. The second problem is to add spare capacities to the physical links to guarantee routability of all logical link demands before and after a physical link failure. We conclude with heuristics that mitigate the computational complexity of the mathematical programming formulations and with simulation results comparing these heuristics with the mathematical programming formulations.**

***Index Terms*—IP-over-WDM; Logical topology routing; Mathematical programming; Survivability.**

I. Introduction

Network survivability is among the most recurring issues when designing telecommunication networks. When a network facility (link or node) fails, a mechanism which guarantees continued network flow and operability is critical. Over the last decade there has been an explosive growth in Internet traffic requiring a high transport capacity of telecommunication networks. While the utilization of wavelength-division multiplexing (WDM) extends the capacity of optical fibers [1], optical fiber failures lead to disruptions in traffic and severe consequences. Ramamurthy *et al.* [2] summarized the protection and restoration mechanisms on WDM networks and examined the routing and wavelength assignment problems. With the development of optical cross-connect (OXC) and optical add–drop multiplexer, WDM is mostly deployed in a point-to-point manner and supports multilayered architectures such as Internet Protocol/Multiprotocol Label Switching (IP/MPLS), Asynchronous Transfer Mode (ATM), and Synchronous Optical Networking/Synchronous Digital Hierarchy (SONET/SDH) [3].

An IP-over-WDM network is a two-layered network where an IP (logical) network is embedded onto a WDM (physical) network. IP routers and OXCs correspond to the logical and physical nodes. Links connecting the nodes in a logical network are called the logical links, and the physical links are realized via optical fibers. The logical nodes are commonly assumed to have corresponding nodes in the physical network. On the other hand, not all physical nodes may exist in the logical network. A router-to-router link is implemented through a wavelength on a path between two end nodes in a WDM network bypassing opto–electro–optic conversions on intermediate nodes in the path. This path is called a *lightpath*. Each optical fiber may carry multiple lightpaths; hence a failure on an optical fiber may have a cascading effect causing failures on multiple logical links, resulting in a huge amount of data traffic (terabits/s) loss. This has given rise to extensive interest in the study of survivability issues in the IP-over-WDM network. In this paper, the terms "mapping" and "routing" will be used interchangeably.

Examples of a survivable mapping and an unsurvivable mapping of the links of a logical topology [Fig. 1(a)] onto the links of a physical topology are shown in Fig. 1. In the mapping of Fig. 1(c), when physical link (4,5) fails, logical links (2,4) and (4,6), whose lightpaths are both routed through physical link (4,5), fail simultaneously, causing the logical

Manuscript received July 11, 2013; revised November 24, 2013; accepted December 1, 2013; published January 29, 2014 (Doc. ID 181689).

Tachun Lin (e-mail: tlin@cameron.edu) is with the Department of Computing and Technology, Cameron University, Lawton, Oklahoma 73505, USA.

Zhili Zhou is with IBM Research Collaboratory, Singapore.

Krishnaiyan Thulasiraman is with the School of Computer Science, University of Oklahoma, Norman, Oklahoma 73019, USA.

Guoliang Xue is with the School of Computing, Informatics, and Decision Systems Engineering, Arizona State University, Tempe, Arizona 85287, USA.

Sartaj Sahni is with the Computer and Information Science and Engineering Department, University of Florida, Gainesville, Florida 32611, USA.

http://dx.doi.org/10.1364/JOCN.6.000190

 © 2014 Optical Society of America

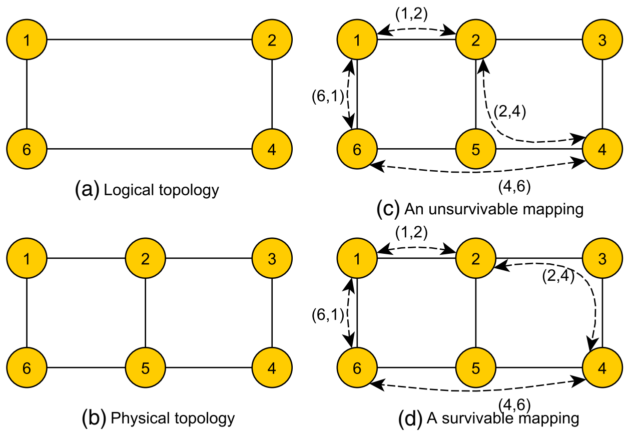

Fig. 1. Unsurvivable and survivable mappings for logical topology.

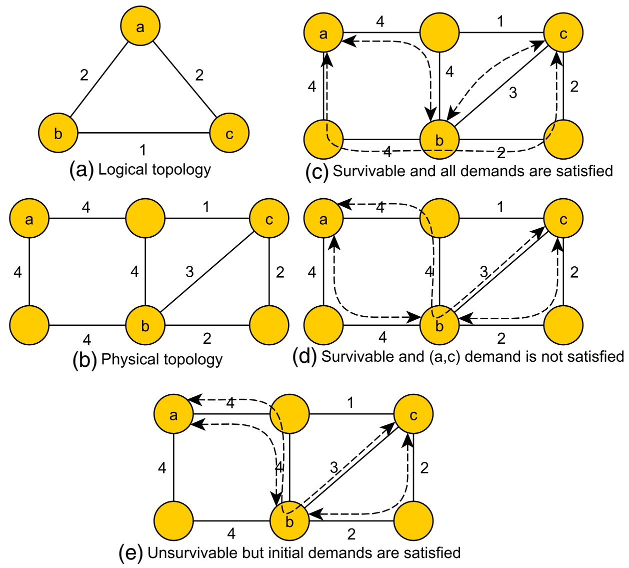

Fig. 2. Capacitated survivability and demand satisfaction against single failure.

topology to become disconnected since logical node 4 is no longer connected to other nodes in the logical topology after this physical link failure. In contrast, in Fig. 1(d) no physical link failure can disconnect the logical topology; hence the mapping is survivable. Therefore, survivability of a mapping can be guaranteed if the lightpaths in the physical topology corresponding to this mapping are all link-disjoint.

Most previous research concentrated on survivable design of uncapacitated IP-over-WDM networks, while in practice physical link capacities and logical link demands are usually considered during the design phase. In the rest of this paper, we consider survivable logical topology design in IP-over-WDM networks with capacity and demand constraints on physical and logical links, respectively. For uncapacitated IP-over-WDM networks, survivability is achieved if the logical network remains connected after any physical link failure. In such a case, since the logical network will be connected after a physical link failure, the existence of alternative lightpaths for the failed logical links is guaranteed. However, if the physical link capacity is taken into consideration, demands on logical links may not be satisfied after physical link failure(s) even if the logical network remains connected. Thus, the original definition of survivability in uncapacitated IP-over-WDM networks does not apply to capacitated networks. In order to satisfy demands on logical links we need to add *spare capacity* to each physical link, which is the extra capacity required to carry the disrupted traffic. Figures 2(a) and 2(b) show a logical network with demands on its links and a physical network with capacities on its links. A survivable routing satisfying both logical link demands and guaranteeing logical graph survivability after a single physical link failure is shown in Fig. 2(c). The mapping in Fig. 2(d) is survivable while (a,c) demand cannot be satisfied under the failure of physical link (b,c). An example of unsurvivable mapping satisfying all initial demands is given in Fig. 2(e).

The contributions of this paper are as follows: 1) this paper first provides definitions and problems of survivable routing in IP-over-WDM networks with physical link capacity and logical link demand; 2) mixed integer linear program (MILP) formulations and heuristic algorithms are developed to solve the problems effectively; and 3) the models and formulations introduced in this paper provide a framework, which can be generalized or extended to formulations for logical topology augmentation for guaranteed survivability, survivability against multiple physical link failures, maximizing minimum cross-layer cut (MCLC), maximizing after-failure connectivity of the logical topology, maximizing logical capacity, and load balancing and energy-minimized survivable routing.

The rest of the paper is organized as follows. Section II provides a review of literature in the related areas. Formal definitions of weak and strong survivability and notations are presented in Section III. Section IV provides MILP formulation of the weak survivability problem with the objective of maximizing the total demand satisfaction under a survivable routing. We propose single-stage and two-stage formulations to achieve the objective by rerouting at the physical layer.

In Section V, a two-stage formulation for strongly survivable routing under minimum spare capacity requirements is developed. To mitigate the computational complexity of the mathematical programming formulations, we provide heuristics in Section VI for weakly survivable and strongly survivable routing problems. Preliminary simulation results comparing these heuristics with the mathematical programming formulations are also given in Section VI.

This paper is an expanded version of our previous work [4].

II. Related Work

As we noted in the previous section, survivability of a logical topology mapping can be guaranteed if the lightpaths in the physical topology corresponding to this mapping are all link-disjoint. Since finding mutually disjoint paths between multiple pairs of nodes is NP-complete [5], survivable design of the logical topology in an IP-over-WDM network is also an NP-complete problem. Modiano and Narula-Tam [6] proved a necessary and sufficient condition for survivable routing under a single physical link failure in IP-over-WDM networks and formulated the problem as an integer linear program (ILP). Todimala and Ramamurthy [7] adapted the concept of a shared risk link group introduced in [8] and also computed the routing through an ILP formulation. Extensions of the work in [6] are given in [9,10]. Reference [9] introduced certain connectivity metrics for layered networks and provided ILP formulations for the lightpath routing problem satisfying these metrics. In particular, they provided approximation heuristics for lightpath routing maximizing the MCLC metric. Kan *et al.* [10] discussed the relationship between survivable lightpath routing and spare capacity requirements on the logical links to satisfy the original traffic demands after failures. A common drawback of ILP approaches is that they are not scalable as the network size increases. Hence, heuristic approaches that provide approximations to the optimal solutions have been considered in the literature.

To handle the drawback of ILP approaches, Kurant and Thiran [11] proposed the survivable mapping by ring trimming (SMART) framework, which first attempts to find link-disjoint mappings for the links of a subgraph (instead of all the links in the original graph) of the given logical graph. Another approach proposed by Lee *et al.* [12] utilized the concept of ear-decomposition on biconnected topologies. One can show that this is, in fact, a special variant of the framework given in [11], though it was developed independently. Javed and co-workers obtained improved heuristics based on SMART [13,14]. Using duality theory in graphs, a generalized theory of logical topology survivability was given by Thulasiraman and co-workers [15,16]. Thulasiraman *et al.* [17] considered the problem of augmenting the logical graph with additional links to guarantee the existence of a survivable mapping. It has been shown in [17] that if the physical network is three-edge connected, an augmentation of the logical topology that is guaranteed to be survivable is always possible. An earlier work that discussed augmentation is [18].

There has been a great deal of research on the single layer network survivability problem, in particular assignment of spare capacities on the physical links to guarantee the required network flows after link failures. Some recent works in this area are [19,20]. Some of the other works that studied the spare capacity assignment problem under survivability requirements are [21,22].

All of these works do not consider the notion of survivability of the logical layer that is critical in IP-over-WDM networks. References [23,24] studied the survivable optical virtual private network (VPN) problems. Reference [23] introduced approaches for resource allocation with static demands in survivable optical networks supporting VPN. Reference [24] defined the resilient VPN design problem minimizing the number of lightpaths required in a VPN. Both of them did not consider adding spare capacity and imposed wavelength sharing constraints. As remarked earlier, Kan *et al.* [10] discussed the relationship between survivable lightpath routing and spare capacity requirements on the logical links to satisfy the original traffic demands after failures. In contrast, in this paper we investigate lightpath routing that maximizes the demand satisfaction of the logical graph before and after a physical link failure, as well as lightpath routing that minimizes spare capacity requirements on the physical links that guarantees strong survivability as defined in Section III.

III. Problem Description and Notations

We use the terms "network" and "topology," "edge" and "link," and "node" and "vertex" interchangeably throughout the paper. Let $G_L = (V_L, E_L)$ be a logical network and $G_P = (V_P, E_P)$ be a physical network in an IP-over-WDM network. Let (i,j) be a physical link and (s,t) be a logical link. Capacity on physical link (i,j) is c_{ij} and demand on logical link (s,t) is d_{st}. We now define the survivability criteria considered in this paper. We assume both the physical and logical networks are at least two-edge connected.

Definition 1: An IP-over-WDM network with logical and physical topologies $G_L = (V_L, E_L)$ and $G_P = (V_P, E_P)$ is *weakly survivable* if, after any physical link failure, G_L remains connected.

Definition 2: An IP-over-WDM network with $G_L = (V_L, E_L)$, $G_P = (V_P, E_P)$, capacity c_{ij} for each physical link (i,j), and demand d_{st} for each logical link (s,t) is *strongly survivable* if, before and after any physical link (i,j) failure, G_L remains connected and d_{st} can be satisfied for all $(s,t) \in E_L$.

Definition 3: The *spare capacity* on a physical link is the extra capacity required to satisfy all d_{st} before and after any (i,j) failure while the logical topology remains connected. Note: If the spare capacity requirement on each physical link is zero after a physical link failure, then the network is strongly survivable.

In this paper, we investigate the following two problems and their extensions.

Problem 1: Determine lightpath routings, for all logical demands, which guarantee weak survivability by rerouting after any physical link (i,j) failure and maximize the sum of satisfied logical demands before and after rerouting with the limitation of physical link capacities.

Problem 2: Determine lightpath routings for logical demands, which guarantee strong survivability under minimum spare capacity requirements.

We first study Problem 1 as a single-stage problem in Subsection IV.A. Due to the scalability and computational complexity of the single-stage approach, we propose a two-stage approach for Problems 1 and 2 and present the formulations for the two-stage problems in Subsections IV.B and V.

IV. Weakly Survivable Routing/Rerouting and Maximum Logical Link Demand Satisfaction

In this section we investigate Problem 1. There are two aspects to the problem, namely, lightpath routing and rerouting that guarantees weak survivability with maximum logical link demand satisfaction before and after any physical link failure. We introduce variables and parameters used in the formulation in Table I. Note that $(i,j),(k,\ell)\in E_P,(s,t)\in E_L,i,j,k,\ell\in V_P,s,t\in V_L$.

A. Single-Stage Solution Approach for Weakly Survivable Routing

We now present a single-stage solution approach for Problem 1 through an MILP formulation, which consists of two groups of constraints addressing weak survivability and demand satisfaction before and after a physical link failure, respectively. Group 1 contains constraints (1) to (10), which 1) generate lightpaths for logical demands, 2) guarantee the same flow (i.e., satisfied demand) along a lightpath, 3) prevent the flow from exceeding the demands and capacities, and 4) ensure survivability against a physical link failure. Under the failure of any physical link, constraints (11) to (23) in Group 2 reroute the disconnected logical links utilizing residual capacities on each physical link and avoiding the rerouted lightpaths from passing through the failed physical link. Note that the residual capacities are determined by the original capacities subtracting the capacities occupied by the working (undisrupted) lightpaths, which restrict the existence of a new lightpath routing. A logical link representing a demand between logical nodes s and t will be denoted by (s,t) if $s<t$, and otherwise by (t,s). First, we formulate constraints in Group 1 as follows:

TABLE I
Parameters and Variables Used in MILP Formulations

Parameter	Description
c_{ij}	Capacity on physical link (i,j)
d_{st}	Demand on logical link (s,t)
M	A very large number
Variable	**Description**
y_{ij}^{st}	Binary variable indicates whether the logical link $(s,t)\in E_L$ is routed through the physical link $(i,j)\in E_P$ by direction from i to j. If yes, $y_{ij}^{st}=1$; otherwise, $y_{ij}^{st}=0$.
f_{ij}^{st}	Flow on physical link (i,j) due to lightpath (s,t).
r_{st}^{ij}	Fractional variable for connectivity constraints. $r_{st}^{ij}=0$ indicates that (s,t) is disconnected if (i,j) fails.
ρ_{st}	The satisfied demand for the logical link (s,t).
λ_{ij}^{st}	Maximal rerouted demand of disconnected logical link (s,t) after physical link (i,j) failure and rerouting.
p_{ij}^{st}	Binary variable indicates whether logical link (s,t) is routed through physical link (i,j); if yes, $p_{ij}^{st}=1$; otherwise, $p_{ij}^{st}=0$.
$x_{k\ell ij}^{st}$	Rerouted flow on (k,ℓ), which is for rerouting disconnected logical link (s,t)'s demand after the physical link (i,j) failure.
$z_{k\ell ij}^{st}$	Binary variable indicates whether lightpath for (s,t) is rerouted through (k,ℓ) after (i,j) failure; if yes, $z_{k\ell ij}^{st}=1$; otherwise, $z_{k\ell ij}^{st}=0$.
$\eta_{k\ell ij}^{st}$	Fractional variable for rerouted flow through (k,ℓ) for disconnected (s,t) after (i,j) failure.

Lightpath constraints:

$$\sum_{(i,j)\in E_P} y_{ij}^{st}-\sum_{(j,i)\in E_P} y_{ji}^{st}=\begin{cases}1, & \text{if } i=s,\\ -1, & \text{if } i=t,\\ 0, & \text{if } i\neq\{s,t\},\end{cases}\qquad i\in V_P,(s,t)\in E_L, \tag{1}$$

$$y_{ij}^{st}+y_{ji}^{st}\le 1,\qquad (i,j)\in E_P,(s,t)\in E_L, \tag{2}$$

$$\sum_{(i,j),(j,i)\in E_P}(y_{ij}^{st}+y_{ji}^{st})\le 2,\qquad i\in V_P,(s,t)\in E_L. \tag{3}$$

Flow conservation constraints:

$$\sum_{(i,j)\in E_P} f_{ij}^{st}-\sum_{(j,i)\in E_P} f_{ji}^{st}=\begin{cases}\rho_{st}, & \text{if } i=s,\\ -\rho_{st}, & \text{if } i=t,\\ 0, & \text{if } i\neq\{s,t\},\end{cases}\qquad i\in V_P,(s,t)\in E_L, \tag{4}$$

$$\rho_{st}\le d_{st},\qquad (s,t)\in E_L. \tag{5}$$

Bounded flow constraint:

$$f_{ij}^{st}\le My_{ij}^{st}\quad\text{and}\quad f_{ji}^{st}\le My_{ji}^{st},(i,j)\in E_P,(s,t)\in E_L. \tag{6}$$

Capacity constraint:

$$\sum_{(s,t)\in E_L}(f_{ij}^{st}+f_{ji}^{st})\le c_{ij},\qquad (i,j)\in E_P. \tag{7}$$

Survivability constraints:

$$\sum_{(s,t)\in E_L} r_{st}^{ij}-\sum_{(t,s)\in E_L} r_{ts}^{ij}=\begin{cases}-1, & \text{if } s=v_1,\\ \frac{1}{|V_L|-1}, & \text{if } s\neq v_1,\end{cases}\qquad v_1\in V_L,(i,j)\in E_P, \tag{8}$$

$$0\le r_{st}^{ij}\le 1-(y_{ij}^{st}+y_{ji}^{st}),\qquad (i,j)\in E_P,(s,t)\in E_L, \tag{9}$$

$$0\le r_{ts}^{ij}\le 1-(y_{ij}^{st}+y_{ji}^{st}),\qquad (i,j)\in E_P,(s,t)\in E_L. \tag{10}$$

Next, we explain the purpose of each constraint.

Lightpath constraints (1) through (3): Constraint (1) with binary variable y_{ij}^{st} provides a lightpath routing for each logical link (s,t) with single unit flow. Physical links for which y_{ij}^{st} are nonzero define the lightpath for the logical link (s,t). Constraint (2) guarantees that a lightpath is not routed through a physical link in its two directions. Constraint (3) eliminates other loops by avoiding a lightpath to revisit the same node on the lightpath. Thus we have the following:

Proposition 1: The lightpath constraints provide lightpaths for $(s,t) \in E_L$ and eliminate cycles.

Flow conservation, bounded flow, and capacity constraints (4) through (7): Constraints (4) and (5) push a flow of value ρ_{st} through the lightpath for logical link (s,t). Bounded flow constraint (6) guarantees that each physical link (i,j) carries flow only if the lightpath(s) is routed through at least one direction of the physical link. Here M is a very large number greater than the maximum link capacity. Capacity constraint (7) requires that the total flow of all the lightpaths carried by each physical link is no more than the corresponding physical link capacity.

Survivability constraints (8) through (10): Based on the lightpaths generated by constraints (1) through (3), after physical link (i,j) failure, if $y_{ij}^{st}+y_{ji}^{st}=1$, the lightpath for logical link (s,t) is disconnected; otherwise, the lightpath for (s,t) remains connected. If the lightpath for logical link (s,t) is routed through (i,j), $r_{st}^{ij}=0$; otherwise, $r_{st}^{ij}\le 1$. Constraints (8) through (10) force a flow of unit value to be pushed to node v_1. The logical links with nonzero r_{st}^{ij} (those links that are not broken) define a connected subgraph of G_L after the physical link (i,j) failure. Therefore, if these constraints have a solution for all physical and logical links, then the selected routing is survivable. Thus the following proposition holds.

Proposition 2: The survivability constraints provide the necessary and sufficient condition for survivable routing in the IP-over-WDM network.

We wish to note that the survivability constraints (8)–(10) were first employed in the paper by Deng *et al.* [25].

Now we study the problem of rerouting for disrupted logical links after each physical link failure. After any physical link (i,j) failure, demands on disconnected lightpaths which are routed through (i,j) by constraints in Group 1 would be rerouted with new ligthpaths through physical links with residual capacities. We formulate the constraints in Group 2 as follows:

Rerouting constraints:

$$\sum_{(k,\ell)\in E_P\setminus\{(i,j)\}} z_{k\ell ij}^{st}-\sum_{(\ell,k)\in E_P\setminus\{(i,j)\}} z_{\ell kij}^{st} = \begin{cases} p_{ij}^{st}, & \text{if } k=s,\\ -p_{ij}^{st}, & \text{if } k=t, \quad k\in V_P, (s,t)\in E_L, (i,j)\in E_P,\\ 0, & \text{if } k\neq\{s,t\},\end{cases} \tag{11}$$

$$z_{k\ell ij}^{st}+z_{\ell kij}^{st}\le 1,\quad (i,j)\in E_P, (k,\ell)\in E_P\setminus\{(i,j)\}, (s,t)\in E_L, \tag{12}$$

$$\sum_{(k,\ell),(\ell,k)\in E_P\setminus\{(i,j)\}} (z_{k\ell ij}^{st}+z_{\ell kij}^{st})\le 2,\quad k\in V_P, (i,j)\in E_P, (s,t)\in E_L, \tag{13}$$

$$z_{k\ell ij}^{st}+z_{\ell kij}^{st}\le y_{ij}^{st}+y_{ji}^{st},\quad (i,j)\in E_P, (k,\ell)\in E_P\setminus\{(i,j)\}, (s,t)\in E_L, \tag{14}$$

$$p_{ij}^{st}\ge y_{ij}^{st}+y_{ji}^{st},\qquad (i,j)\in E_P, (s,t)\in E_L. \tag{15}$$

Rerouted flow conservation constraints:

$$\sum_{(k,\ell)\in E_P\setminus\{(i,j)\}} x_{k\ell ij}^{st}-\sum_{(\ell,k)\in E_P\setminus\{(i,j)\}} x_{\ell kij}^{st} = \begin{cases} \lambda_{ij}^{st}, & \text{if } k=s,\\ -\lambda_{ij}^{st}, & \text{if } k=t, \quad k\in V_P, (i,j)\in E_P, (s,t)\in E_L,\\ 0, & \text{if } k\neq\{s,t\},\end{cases} \tag{16}$$

$$\lambda_{ij}^{st}\le d_{st},\qquad (s,t)\in E_L, (i,j)\in E_P. \tag{17}$$

Residual capacity constraints:

$$\sum_{(s,t)\in E_L} (x_{k\ell ij}^{st}+x_{\ell kij}^{st})\le c_{k\ell}-\sum_{(s,t)\in E_L} (\eta_{k\ell ij}^{st}+\eta_{\ell kij}^{st}),\quad (i,j),(k,\ell)\in E_P, (k,\ell)\neq(i,j), \tag{18}$$

$$x_{k\ell ij}^{st}\le M z_{k\ell ij}^{st},\quad x_{\ell kij}^{st}\le M z_{\ell kij}^{st},\quad (s,t)\in E_L, (i,j)\in E_P, (k,\ell)\in E_P\setminus\{(i,j)\}. \tag{19}$$

Capacity reservation constraints:

$$\eta_{k\ell ij}^{st}\le M[1-(y_{ij}^{st}+y_{ji}^{st})], (i,j),(k,\ell),(\ell,k)\in E_P,\quad (k,\ell),(\ell,k)\neq(i,j), (s,t)\in E_L, \tag{20}$$

$$\eta_{k\ell ij}^{st}\ge f_{k\ell}^{st}-M(y_{ij}^{st}+y_{ji}^{st}), (i,j),(k,\ell),(\ell,k)\in E_P,\quad (k,\ell),(\ell,k)\neq(i,j), (s,t)\in E_L, \tag{21}$$

$$\eta_{k\ell ij}^{st}\le f_{k\ell}^{st}+M(y_{ij}^{st}+y_{ji}^{st}), (i,j),(k,\ell),(\ell,k)\in E_P,\quad (k,\ell),(\ell,k)\neq(i,j), (s,t)\in E_L, \tag{22}$$

$$\eta_{k\ell ij}^{st}, \lambda_{ij}^{st}, x_{k\ell ij}^{st}\ge 0, z_{k\ell ij}^{st}\in\{0,1\},\quad (s,t)\in E_L, (i,j)\in E_P, (k,\ell)\in E_P\setminus\{(i,j)\}. \tag{23}$$

Combining the constraints in Groups 1 and 2 with an objective to maximize the sum of satisfied logical demands based on routing and rerouting before and after any physical link failure, a single-stage approach for Problem 1 can now be formulated as follows in Algorithm 1.

Algorithm 1 Single-Stage Weakly Survivable Routing Design Under Capacity Constraints (WSRD-CC-SS)

Objective: maximizing satisfied demands before and after failure

$$\max \sum_{(s,t)\in E_L} \rho_{st} + \sum_{(s,t)\in E_L} \sum_{(i,j)\in E_P} \lambda_{ij}^{st}$$

s.t. Constraints (1) to (23)

Rerouting and rerouted flow conservation constraints (11) through (17): Constraints (11) through (13) [(similar to constraints (1) through (3)] provide new lightpaths for logical links which are broken after (i,j) failure. Constraint (14) restricts that the rerouted lightpath for logical link (s,t) would only be generated if its original lightpath has been routed through the failed physical link (i,j). Constraint (15) guarantees that a rerouted lightpath is generated only for logical link (s,t) whose lightpath is routed through the failed physical link (i,j). Constraints (16) and (17) [similar to constraints (4) and (5)] determine the rerouted demand of logical link (s,t) after physical link (i,j) failure and force the rerouted demand for (s,t) to be bounded by the demand of logical link (s,t).

Residual capacity constraints (18) and (19): Constraint (18) ensures that demands on failed logical links can only be (partially) satisfied by physical links with residual capacities. Constraint (19) [similar to constraint (6)] forces the rerouted flows to be zero for all links that are not used by rerouted lightpaths.

Capacity reservation constraints (20) through (23): Constraint (20) ensures that if the original lightpath of (s,t) is routed through the failed physical link (i,j), the capacity of physical link (k,ℓ) consumed by the flow of original (s,t)'s lightpath would be released and utilized by rerouted lightpaths. Constraints (21) and (22) together preserve the capacity of physical link (k,ℓ) utilized by the demand of (s,t)'s lightpath, which remains connected after physical link (i,j) failure. Constraints (20) to (22) guarantee that $\eta_{k\ell ij}^{st} = f_{k\ell}^{st}$ if $y_{ij}^{st} = y_{ji}^{st} = 0$; otherwise, $\eta_{k\ell ij}^{st} = 0$. Constraint (23) provides the feasible region for fractional and binary variables.

B. Two-Stage Approach for Weakly Survivable Routing

The single-stage formulation presented in Subsection IV.A provides an overall solution which not only generates lightpath routings before and after any physical link failure but also guarantees that the sum of satisfied demands is maximal with the capacity constraints. However, this approach suffers from a scalability issue where the number of constraints and variables involved would increase exponentially given larger logical and physical topologies (demonstrated in Section VI). Hence, we propose to solve the weakly survivable routing problem in two stages to overcome this drawback.

Stage 1: Determine a lightpath routing that guarantees weak survivability and maximizes the sum of logical demands satisfied by this routing (i.e., maximum logical demand satisfaction) before a physical link failure.

Stage 2: Based on optimal results from Stage 1, after any physical link (i,j) failure, determine a rerouting for each disconnected logical link which avoids (i,j) and maximizes the sum of logical demands utilizing the residual capacity. Note that demands and routings of the logical links not affected by (i,j) failure still occupy physical capacities along their lightpaths.

We now present an MILP formulation (called the WSRD-CC algorithm) of the first stage of Problem 1.

Algorithm 2 WSRD-CC (First Stage of Problem 1)

MILP formulation for the first stage of the weakly survivable routing design
(Objective: maximizing total satisfied logical demand):

$$\max \sum_{(s,t)\in E_L} \rho_{st} \tag{24}$$

s.t. Constraints (1)–(10),
$y_{ij}^{st} \in \{0,1\}, r_{ij}^{st}, f_{ij}^{st}, \rho_{st} \geq 0, (s,t) \in E_L, (i,j) \in E_P$

There are different ways to evaluate the largest satisfied demand on logical links. For the first stage of Problem 1, we aim to maximize the total satisfied demands in the logical network in constraint (24).

With the WSRD-CC algorithm, we obtain the survivable lightpath routing information $y_{ij}^{st^*}$ and the corresponding satisfied logical demand ρ_{st}^*. Note that the constraint set in the first-stage problem is the same as Group 1's constraints in the single-stage formulation. We next consider the second stage of Problem 1.

Once a physical link (i,j) fails, we need to reroute lightpaths that were routed through link (i,j) to satisfy at least partially the original demands on these lightpaths. With $y_{ij}^{st^*}$, we know that if $y_{ij}^{st^*} = 1$, then the lightpath of logical link (s,t) is routed through (i,j). Thus, for a given (i,j), we only need to reroute lightpaths of logical links that are in the set $R_{ij} = \{(s,t){:}y_{ij}^{st^*} + y_{ji}^{st^*} = 1, (s,t) \in E_L\}$. Therefore, in the second stage, after any physical link (i,j) failure, the disrupted network flow is rerouted through a new lightpath going through physical links with residual capacities (the residual capacity on physical links after any physical link failure). The existence of a rerouted lightpath is restricted by the residual capacities. We formulate the second-stage constraints as follows.

Rerouting constraints in the second stage problem:

$$\sum_{(k,\ell)\in E_P\setminus\{(i,j)\}} z_{k\ell ij}^{st} - \sum_{(\ell,k)\in E_P\setminus\{(i,j)\}} z_{\ell kij}^{st} = \begin{cases} 1, & \text{if } k=s, \\ -1, & \text{if } k=t, \\ 0, & \text{if } k\neq\{s,t\}, \end{cases} \quad k\in V_P, (s,t)\in R_{ij}, (i,j)\in E_P, \tag{25}$$

$$z_{k\ell ij}^{st} + z_{\ell kij}^{st} \leq 1, (k,\ell) \in E_P\setminus\{(i,j)\}, (s,t) \in R_{ij}, (i,j) \in E_P, \tag{26}$$

$$\sum_{(k,\ell),(\ell,k)\in E_P\setminus\{(i,j)\}} (z^{st}_{k\ell ij}+z^{st}_{\ell kij})\le 2,\quad k\in V_P,(s,t)\in R_{ij},(i,j)\in E_P. \tag{27}$$

Residual capacity constraints in the second-stage problem:

$$\sum_{(s,t)\in R_{ij}} (x^{st}_{k\ell ij}+x^{st}_{\ell kij})\le c_{k\ell}-\sum_{(u,v)\in E_L\setminus R_{ij}} \rho^*_{uv}(y^{uv*}_{k\ell}+y^{uv*}_{\ell k}),$$
$$(k,\ell)\in E_P\setminus\{(i,j)\},(i,j)\in E_P, \tag{28}$$

$$x^{st}_{k\ell ij}\le Mz^{st}_{k\ell ij},$$
$$x^{st}_{\ell kij}\le Mz^{st}_{\ell kij},(s,t)\in R_{ij},(k,\ell)\in E_P\setminus\{(i,j)\},(i,j)\in E_P, \tag{29}$$

$$\sum_{(k,\ell)\in E_P\setminus\{(i,j)\}} x^{st}_{k\ell ij}-\sum_{(\ell,k)\in E_P\setminus\{(i,j)\}} x^{st}_{\ell kij}=\begin{cases}\lambda^{st}_{ij}, & \text{if } k=s,\\ -\lambda^{st}_{ij}, & \text{if } k=t,\\ 0, & \text{if } k\ne\{s,t\},\end{cases}$$
$$k\in V_P,(s,t)\in R_{ij},(i,j)\in E_P, \tag{30}$$

$$\lambda^{st}_{ij}\le d_{st},\qquad (s,t)\in R_{ij},(i,j)\in E_P. \tag{31}$$

Algorithm 3 Weakly Survivable Routing Design with Maximum Logical Capacity (MAXCAP-WSRD) (Second Stage of Problem 1)

MILP formulation for the weak survivability design (Objective: maximizing logical demand satisfaction)

$$\max \sum_{(i,j)\in E_P}\sum_{(s,t)\in R_{ij}} \lambda^{st}_{ij} \tag{32}$$

s.t. Constraints (25)–(31)

$$\eta^{st}_{k\ell ij},\lambda^{st}_{ij},x^{st}_{k\ell ij}\ge 0, z^{st}_{k\ell ij}\in\{0,1\},(s,t)\in R_{ij},$$
$$(i,j)\in E_P,(k,\ell)\in E_P\setminus\{(i,j)\}. \tag{33}$$

The goal for algorithm MAXCAP-WSRD, the MILP formulation of the second stage of Problem 1, is to maximize the total fulfilled disrupted logical demands through rerouted lightpaths after any physical link failure.

Rerouting constraints in the second stage problem (25)–(27): With R_{ij} generated in WSRD-CC, constraints (25) to (27) generate new lightpaths avoiding (i,j) for all broken logical links.

Residual capacity and flow equivalence constraints in the second-stage problem (28)–(31): Constraint (28) restricts the total rerouted flow on physical link (k,ℓ) to be within its residual capacity. With the optimal routings from the first-stage problem, the residual capacity of physical link (k,ℓ) is $c_{k\ell}-\sum_{(u,v)\in E_L\setminus R_{ij}}\rho^*_{uv}(y^{uv*}_{k\ell}+y^{uv*}_{\ell k})$. Here the second term at the right-hand side of constraint (28) is the total flow on link (k,ℓ) due to all the flows contributed by unbroken lightpaths after failure of (i,j). Constraint (29) [similar to constraint (6)] forces flows to be zero on all links that are not in a rerouted lightpath. Constraints (30) and (31) push a flow of value λ^{st}_{ij} on the selected lightpath for logical link (s,t) after physical link (i,j) failure.

V. Strongly Survivable Lightpath Routing Under Minimum Physical Spare Capacity

In this section, we study the strongly survivable routing problem by allocating the spare capacity to satisfy all logical demands before and after physical link failure. There are two aspects to the capacity allocation problem: 1) spare capacity allocation on the logical links and 2) spare capacity allocation on the physical links. Reference [26] studied protection at the WDM layer, (i.e., set up a backup lightpath for every primary lightpath and the corresponding maximum capacity allocation). Reference [10] studied spare capacity allocation on the logical links. In the IP-over-WDM network, the main capacity restriction is from physical links (i.e., the bandwidths on fibers.) Compared with spare capacity allocation in the logical network, the spare capacity allocation in the physical network reflects real insufficient capacity which restricts demand satisfaction because the capacity on the logical link is an estimation based on capacities on physical links. We now investigate Problem 2, which requires satisfaction of all logical demands before and after any physical link failure with additional capacities added to some of the physical links when necessary. Our objective is to minimize the total spare capacity added. With the same concerns on scalability and computational complexity issues of the single-stage solution approach, we proceed toward the above objective in two stages as follows.

Stage 1: We first determine the survivable routing that satisfies all logical demands before any physical link failure. This may require adding spare capacities on physical links. Our objective is to minimize the total spare capacity requirement.

The MILP for the first stage of Problem 2 is the same as the WSRD-CC algorithm except that we use d_{st} instead of ρ_{st} in Eq. (4) and rewrite Eq. (7) as

$$\sum_{(s,t)\in E_L} (f^{st}_{ij}+f^{st}_{ji})\le c_{ij}+\eta^{(1)}_{ij},\quad (i,j)\in E_P. \tag{34}$$

Here $\eta^{(1)}_{ij}$ is a newly introduced variable that represents the spare capacity to be added to physical link (i,j). We modify the objective to minimize the total spare capacity:

$$\min \sum_{(i,j)\in E_P} \eta^{(1)}_{ij}. \tag{35}$$

Stage 2: With the information on optimal lightpath routing and required spare capacity obtained from Stage 1, we determine the total spare capacity requirement to satisfy all logical demands after any physical link failure by solving the following MILP formulations.

First, we introduce two auxiliary variables $\eta^{ij}_{k\ell}$ and $\eta^{(2)}_{k\ell}$, which represent spare capacity for a rerouted lightpath after physical link (i,j) failure and the maximal spare capacity added on an undisrupted physical link for rerouted lightpaths after any (i,j) failure. The objective of the second-stage problem is to minimize the sum of spare capacities allocated for rerouted lightpaths after any (i,j) failure.

Algorithm 4 MILP for Stage 2 of Strongly Survivable Routing

$$\min \sum_{(i,j)\in E_P} \eta^{(2)}_{ij}$$

s.t. Constraints (25) to (27),

$$\sum_{(s,t)\in R_{ij}} d_{st}(z^{st}_{k\ell ij}+z^{st}_{\ell kij}) \le c_{k\ell}+\eta^{(1)}_{k\ell} - \sum_{(s,t)\in E_L\backslash R_{ij}} d_{st}(y^{st}_{k\ell}+y^{st}_{\ell k}) + \eta^{ij}_{k\ell},$$
$$(i,j)\in E_P, (k,\ell)\in E_P\backslash\{(i,j)\}, \quad (36)$$

$$\eta^{(2)}_{k\ell} \ge \eta^{ij}_{k\ell}, \qquad (i,j)\in E_P, (k,\ell)\in E_P\backslash\{(i,j)\}. \quad (37)$$

Constraint (36) guarantees that extra spare capacity $\eta^{ij}_{k\ell}$ could be allocated for rerouted lightpaths after physical link (i,j) failure. The left-hand side represents the total flow on physical link (k,ℓ) due to rerouting of all logical links in R_{ij}, and the term on the right-hand side is the sum of residual and spare capacity available on (k,ℓ) after (i,j)'s failure. $\eta^{(2)}_{k\ell}$ is then the maximum of all $\eta^{ij}_{k\ell}$'s due to all physical link failures. Constraint (37) calibrates the maximal second-stage spare capacity on each physical link after any physical link failure. Note that for each physical link the sum of the spare capacity calculated in stage 1 and the spare capacity calculated in stage 2 is the total spare capacity to be added to the link.

We wish to note that the strongly survivable case can also be solved in a single-stage formulation similar to the weakly survivable case.

VI. Heuristics and Experimental Evaluation

The MILP formulations presented in the previous sections require high execution times, though they give optimal values. To mitigate the computational complexity of these formulations, we present, in this section, heuristics for Problem 1 of Section IV, namely, weakly survivable routing maximizing total demand satisfaction, and Problem 2 of Section V for strongly survivable routing with minimum spare capacity requirements.

A. Heuristics for Problem 1 of Section IV

Stage 1: This stage has two steps. In step 1 (lines 1–26), a survivable routing for the given logical topology is generated through augmentation with additional links, if necessary. The lightpath corresponding to each link in the network is also generated in this step. This procedure is summarized in Algorithm 5. In step 2 (lines 27 and 28), flows are pushed along the chosen lightpaths without violating the capacity limits of the physical links. The goal is to achieve a feasible set of flows for the lightpaths that achieve a high demand satisfaction before a physical link failure. An outline of step 1, as described in Algorithm 5, is as follows.

We pick a logical node with the maximum degree as the datum node denoted as Δ. Then we pick any logical node v with degree $\ge$2, map two of v's adjacent edges into disjoint paths in the physical topology, and remove v from the logical topology. This procedure is repeated until no logical node with degree $\ge$2 is left. Next, we pick a node v from the remaining logical topology with degree $=1$ [an edge (u,v)], add a parallel edge $(u,v)'$ to (u,v), and then find disjoint mappings for (u,v) and $(u,v)'$ in the physical topology. Then we remove v from the logical topology. This procedure is executed until all logical nodes with degree $=1$ are eliminated. If after the previous steps there exist nodes v with degree $=0$, we add two parallel edges connecting v and Δ and map them disjointly in the physical topology. During the lightpath generation, a weighted function is applied on physical links, which minimizes the overlapping among all lightpaths. Proof of correctness of logical topology augmentation can be found in [17].

In step 2, we sort the lightpaths by their length, and a unit flow is pushed iteratively along the lightpath corresponding to each logical link $(s,t)\in E_L$ until no more flow can be further pushed on the lightpaths. Here the lightpath with the shortest length would be chosen first. This process will provide an initial set of flows for all logical links. Thus at the end of the first stage we will have a logical topology that has a survivable lightpath routing, while most likely only partial demands would be satisfied. Though augmented logical links do not have demands, they provide the paths needed to keep the logical topology connected after a physical link failure.

Algorithm 5 Stage I. Survivable Lightpath Generation by Augmentation

Input: Physical topology $G_P=(V_P,E_P)$, logical topology $G_L=(V_L,E_L)$, G_P and G_L are at least two-edge connected, lightpath routing M_{G_L}, physical link weight $w[k]$, $k\in E_P$

Output: A survivable augmented logical topology and routing

1: Find datum node $\Delta\in V_L$ with the largest degree
2: $M_{G_L}=\varnothing$, $G'_L=G_L$
3: **while** there exists a node $v\in G'_L$ with degree $\ge$2 **do**
4: Find the minimum weight disjoint lightpaths $p^e, p^{e'}$ for two adjacent edges e, e' of v in G_P
5: Update the routing in M_{G_L}
6: **for** each physical link $k\in p^e\cup p^{e'}$ **do**

$w[k] = w[k] + 1$
end for
$G'_L = G'_L \backslash \{v\}$
end while
while there exists a node $v \in G'_L$ with degree $= 1$ (individual edge $e \in E_L$) **do**
Add a parallel edge e' to e and find the minimum weight disjoint routings for e and e' in G_P
Update the routings in M_{G_L}
for each physical link $k \in p^e \cup p^{e'}$ **do**
$w[k] = w[k] + 1$
end for
$G'_L = G'_L \backslash \{e\}$
end while
while there exists an isolated node $v \in G'_L$ **do**
Add two edges connecting v and Δ and find two minimum weight disjoint routings for the two edges.
Update the routings in M_{G_L}
for each physical link $k \in p^e \cup p^{e'}$ **do**
$w[k] = w[k] + 1$
end for
Remove v from G'_L
end while
Sort $p^e, e \in E_L$ by length
Push unit flow for the demand of each $e, e \in E_L$ iteratively (demand with the shortest lightpath first) until no further demand can be pushed.

Stage 2: In this stage we take down each physical link (i,j) (representing the link failure) one at a time. Let R_{ij} be the set of disconnected logical links due to the failure of physical link (i,j). We then calculate the residual capacity available on each physical link after the failure of (i,j). For each logical link in R_{ij} we find a new lightpath with the largest residual capacity while avoiding the physical link (i,j). If the logical demand can be satisfied by the new lightpath, the demand is subtracted from the capacity of the links on the lightpath, and this demand (s,t) is marked as fully satisfied after failure. Otherwise, we calculate the largest possible demand which can be satisfied, push that as flow on the lightpath, and subtract it from the capacity of physical links on the lightpath. Every time, we calculate this new logical demand and recalculate the residual capacities on the physical links in the selected lightpath. These steps are summarized in Algorithm 6.

Algorithm 6 Stage II. Find Maximal Demand Satisfaction

Input: Physical topology $G_P = (V_P, E_P)$, logical topology $G_L = (V_L, E_L)$, G_P and G_L are at least two-edge connected
Output: ϑ_d, the total maximal demand satisfaction after failure
$\vartheta_d = 0$
Find residual physical link capacity on G_P for the current lightpath routing
for each physical link $e_f \in E_P$ **do**
Find logical links L_f whose lightpaths are routed through e_f
Return to the physical links capacities utilized by L_f
for each logical link $e_\ell \in L_f$ **do**
Find an alternative lightpath in G_P with the largest residual capacity without going through e_f
end for
Find the maximal demand satisfaction $\mathcal{D}_s$
$\vartheta_d = \vartheta_d + \mathcal{D}_s$
end for

This process is repeated for all physical links. The sum of all the logical demands satisfied is the total demand satisfaction.

The complexity of the heuristics is dominated by the algorithm to find minimum weight disjoint lightpaths, which has complexity $\mathcal{O}(|E_P| \log_{(|V_P|+|E_P|)} |V_P|)$ [27,28]. To update the flow iteratively for each unit of logical demand, it takes $\mathcal{O}(|E_P| \sum_{(s,t)\in E_L} d_{st})$. Overall, the complexity for the heuristic is $\max\{\mathcal{O}(|V_L||E_P| \log_{(|V_P|+|E_P|)} |V_P|), \mathcal{O}(|E_P| \sum_{(s,t)\in E_L} d_{st})\}$.

B. Heuristics for Problem 2 of Section IV

Stage 1 of this problem is similar to stage 1 of Problem 1 except that the spare capacities are added to satisfy all demands.

Stage 2 of Problem 2: In this stage we take down each physical link (i,j) one at a time. Using the information about the lightpaths generated in stage 1, we calculate the residual capacities available on the physical links after the failure of (i,j). For each failed logical link (s,t), we find a new lightpath that avoids the physical link (i,j). We choose the new lightpath which has the maximum residual capacity and record the extra capacities required on the physical links to satisfy d_{st} if d_{st} is larger than the residual capacity on the chosen lightpath.

This process is repeated for each physical link. The maximum of the spare capacities required for a physical link at the end of these steps is the spare capacity requirement for this link to guarantee strongly survivable routing.

The complexity of the second-stage heuristic is similar to the complexity of the first-stage heuristic.

C. Simulation Results

We now present the environment setting and simulation results for both MILP formulations and heuristic algorithms. First, we present our experimental design. We selected two sets of topologies as the testing cases. The first set of topologies contains networks introduced in [29,30] as physical topologies, shown in Figs. 3–7. The second set of topologies were randomly generated three-edge connected physical topologies with 50, 60, and 70 nodes. Their corresponding logical topologies were chosen to be three-edge connected topologies whose logical nodes were subsets of physical nodes ($|V_L| = \lfloor 0.5 * |V_P| \rfloor$). Tables II and III summarize the information about physical and logical topologies for Figs. 3–7. In all testing cases, capacities and demands assigned to physical and logical links were randomly generated following uniform distributions in [1,50] and [1,100], respectively.

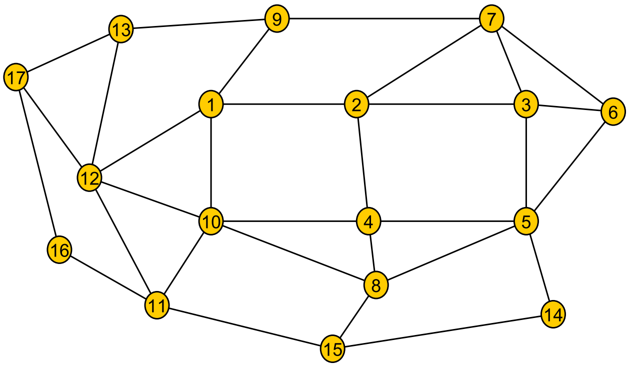

Fig. 3. G6 [29].

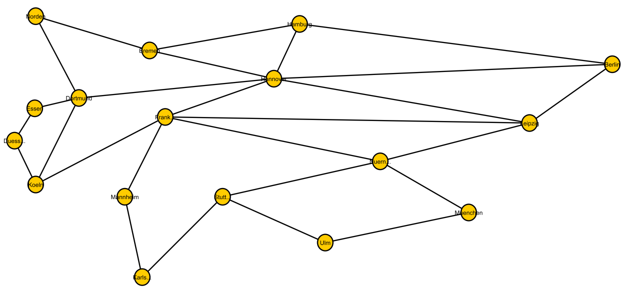

Fig. 4. NOBEL-GERMANY [30].

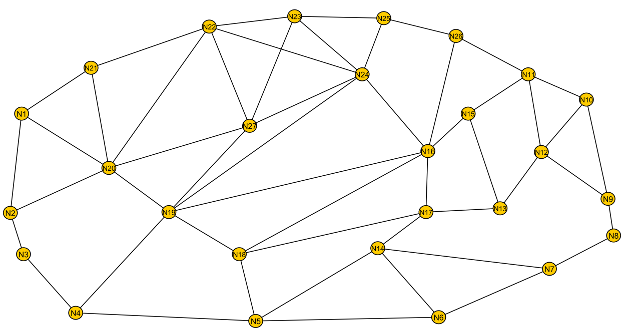

Fig. 5. Norway [30].

We used CPLEX 12.3 to run the weakly and strongly survivable MILP formulations on a machine with a quad-core (with hyperthreading) AMD Opteron processor and 32 GB memory. We assigned a single thread to solve each MILP program and limited the total execution time to be 5 h. The heuristics were implemented using the LEMON library [27], which also utilized a single thread during execution.

Given physical and logical topologies, the scalability of the single-stage and two-stage MILP formulations for the weakly survivable routing problem are presented in Table IV. Note that "WeaklySingleStage," "WeaklyStage1," and "WeaklyStage2" denote the single-stage problem and the first and second stages of the two-stage problem, respectively, for weakly survivable routing. "Variable #" and "Constraint #" represent the number of variables and constraints of the MILP formulations.

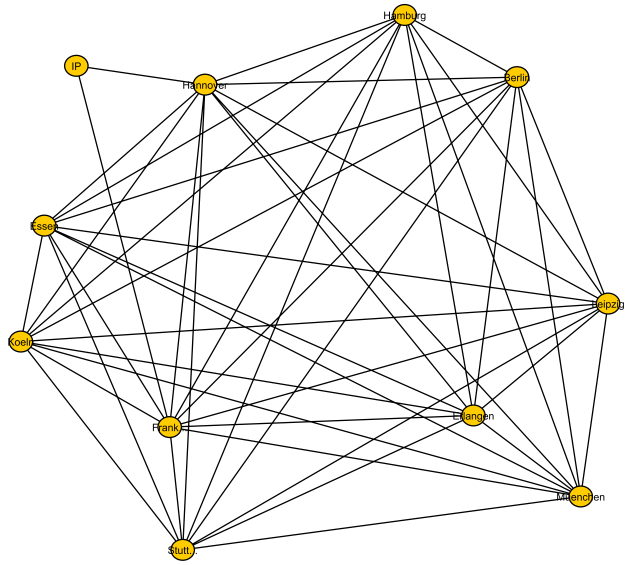

Fig. 6. DFN [30].

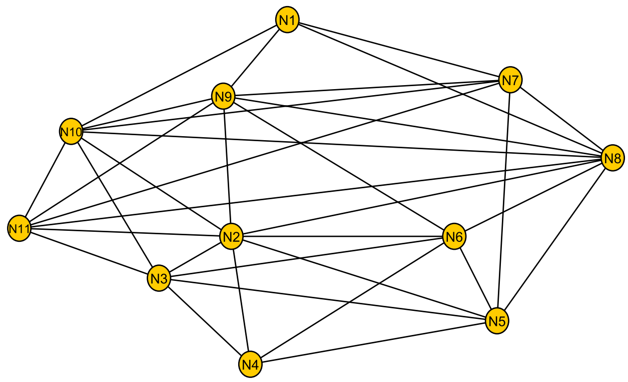

Fig. 7. PDH [30].

TABLE II
PHYSICAL TOPOLOGIES INFORMATION

	Nodes	Edges	MinDeg	MaxDeg	AvgDeg
G6	17	31	2	5	3.64
NOBEL-Germany	17	26	2	6	3.06
Norway	27	51	2	6	3.78
DFN	11	47	2	10	8.55
PDH	11	34	4	8	6.18

TABLE III
LOGICAL TOPOLOGIES INFORMATION

	Nodes	Edges	MinDeg	MaxDeg	AvgDeg
G6	8	12	3	3	3.00
NOBEL-Germany	8	12	3	3	3.00
Norway	13	20	3	4	3.77
DFN	5	8	3	4	3.20
PDH	5	8	3	4	3.20

TABLE IV
SCALABILITY OF MILP FORMULATIONS FOR WEAK SURVIVABILITY

	Variable #	Constraint #
WeaklySingleStage	$\mathcal{O}(\vert E_L\vert\vert E_P\vert^2)$	$\mathcal{O}(\vert E_P\vert(\vert E_P\vert+\vert V_P\vert))$
WeaklyStage1	$\mathcal{O}(\vert E_L\vert\vert E_P\vert)$	$\mathcal{O}(\vert E_L\vert(\vert E_P\vert+\vert V_P\vert)+\vert E_P\vert\vert V_P\vert)$
WeaklyStage2	$\mathcal{O}(\vert E_L\vert\vert E_P\vert^2)$	$\mathcal{O}\left(\vert E_L\vert\vert E_P\vert^2 + \sum_{(i,j)\in E_P} R_{ij}(\vert E_P\vert+\vert V_P\vert)\right)$

TABLE VI
SCALABILITY OF MILP FORMULATIONS FOR STRONG SURVIVABILITY

	Variable #	Constraint #
StrongStage1	$\mathcal{O}(\vert E_L\vert\vert E_P\vert)$	$\mathcal{O}(\vert E_L\vert(\vert E_P\vert+\vert V_P\vert+\vert E_P\vert\vert V_P\vert))$
StrongStage2	$\mathcal{O}(\vert E_L\vert\vert E_P\vert^2)$	$\mathcal{O}\left(\vert E_L\vert\vert E_P\vert^2 + \sum_{(i,j)\in E_P} R_{ij}(\vert E_P\vert+\vert V_P\vert)\right)$

TABLE V
COMPUTATIONAL TIME AND PERFORMANCE OF MILP FORMULATIONS FOR WEAK SURVIVABILITY

	WeaklySingleStage		WeaklyStage1		WeaklyStage2	
	Time	OptGap	Time	OptVal	Time	OptVal
G6	5 h	—	12.18	233	0.15	713
NOBEL-Germany	5 h	0.12%	13.75	241	0.01	849
Norway	5 h	—	5 h	10.75%	154.3	1464
DFN	5 h	13.51%	1.04	209	0.01	209
PDH	5 h	2.49%	6.36	240	0.05	781

TABLE VII
COMPUTATIONAL TIME AND PERFORMANCE OF MILP FORMULATIONS FOR STRONG SURVIVABILITY

	StronglyStage1		StronglyStage2	
	Time	OptVal	Time	OptVal
G6	12.3	12	17.7	119
NOBEL-Germany	13.9	61	22.1	411
Norway	5 h	10.75%	3755	371
DFN	1.15	0	1.28	0
PDH	6.5	0	12.7	49

Table V demonstrates the performance of the single-stage and two-stage problems for weakly survivable routing by computational time (in seconds) and optimal value and optimality gap. If no optimal solution is obtained within the time limit (5 h), we report the optimality gap from CPLEX. In Table V, "OptGap" and "OptVal" denote the optimality gap and optimal objective value of MILP formulations. For the single-stage approach for weakly survivable routing, no optimal solution could be obtained within 5 h. The average optimality gap is 5.37% for our experiments, but CPLEX could not report the optimality gap for testing cases "Norway" and "G6." According to the experimental results, the number of constraints and variables, connectivity of topologies, and node degrees together form the decision factors for the execution time. By solving the first-stage problem, survivable routings were generated for all testing cases, which fulfilled 90.85% of demands on average.

The strongly survivable routing problem could also be solved with a single-stage formulation. However, after comparing the number of constraints, variables, and computation time of MILP formulations for single-stage and two-stage approaches, we decided to proceed with the two-stage approach for the strongly survivable routing problem.

Table VI presents the scalability of the first-stage and second-stage MILP formulations for the strongly survivable routing problem, which is similar to the number for corresponding formulations for weak survivability. Table VII provides the computational time (in seconds) and shows that the spare capacity required is about 0%–4% of the total physical capacity before the failure of any physical link. The after-failure spare capacity required is about 9.5% of the total capacities on average.

TABLE VIII
COMPARISON OF MILP AND HEURISTIC RESULTS ON DEMAND SATISFACTION AFTER FAILURE (WEAKLY SURVIVABLE)

		MILP		Heuristic	
	Demand	dGap	Time	dGap	Time
G6	245	95.10%	12.33	78.13%	0.09
NOBEL-Germany	302	79.80%	13.76	70.89%	0.04
Norway	387	79.33%	5+ h	63.15%	2.12
DFN	209	100%	1.05	86.91%	0.17
PDH	240	100%	6.41	100%	0.10

TABLE IX
COMPARISON OF MINIMUM SPARE CAPACITY REQUIREMENT CALCULATED BY MILP AND HEURISTICS (STRONGLY SURVIVABLE)

		MILP		Heuristic	
	Capcity	cGap	Time	cGap	Time
G6	1989	10.15%	30.0	18.60%	0.38
NOBEL-Germany	1526	17.30%	36.0	20.57%	0.90
Norway	3059	19.64%	5 h	29.22%	1.68
DFN	2550	3.41%	2.43	4.07%	0.08
PDH	1930	0%	19.2	0	0.07

In Tables VIII and IX, "dGap" denotes the satisfied demands over total demands for the weak survivability problem, and "cGap" represents the spare capacity over total capacities for the strong survivability problem. If not specified, the computational time is presented in seconds. For the weakly survivable routing problem, after rerouting,

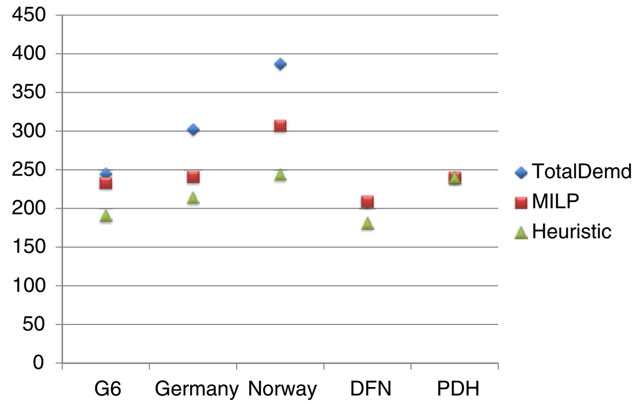

Fig. 8. Comparison of demand satisfaction for weak survivability (after failure).

the two-stage MILPs could satisfy 90.85% of total demands on average; the heuristic algorithm could achieve 79.82% on average. The average gap between the heuristic algorithm and MILPs is 11.03%, but the computation time of the heuristic algorithm is significantly less than the two-stage MILP approach, especially when the size of the topologies is larger. For the strongly survivable routing problem, MILPs and the heuristic algorithm achieve dGaps of 10.1% and 14.49% on average, respectively. Average spare capacities generated by heuristics are about 4.39% more than those of MILPs. We also illustrate the comparison for satisfied demands and added spare capacities in Figs. 8 and 9, respectively.

To further analyze the scalability issue in MILP, we generated larger topologies (shown in Table X) and compared the computation time (in seconds) between the two-stage weakly survivable routing approach and heuristics. While the two-stage MILPs requires 1 to more than 5 h to generate the solution, the heuristic algorithms could still efficiently solve the problem within a short time.

Our results in Tables V and VII–X and in Figs. 8 and 9 demonstrate that our heuristic algorithms can produce solutions with reasonable gaps without enumerating all possible lightpath routings for both weakly and strongly survivable routing problems.

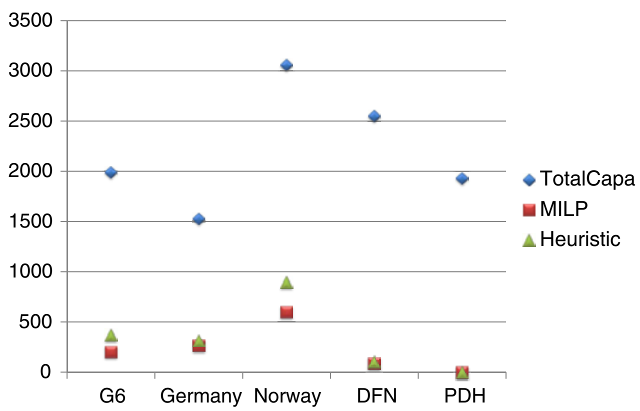

Fig. 9. Comparison of total spare capacities for strong survivability (after failure).

TABLE X
COMPUTATIONAL RESULTS OF MILPS AND HEURISTICS FOR LARGE SIZE PROBLEM

	Physical		Logical		Time	
	Nodes	Edges	Nodes	Edges	MILPs	Heuristic
L1	50	75	25	38	1869.7	11.88
L2	60	90	30	45	3074.7	20.28
L3	70	105	35	53	5 h	36.25

VII. SUMMARY AND CONCLUSIONS

In this paper we presented a comprehensive treatment of mathematical programming frameworks for the survivable logical topology routing problem in capacitated IP-over-WDM optical networks. Under the assumption that both physical and logical topologies are at least two-edge connected and the after-failure rerouting is done at the physical layer, we defined the concepts of weak and strong survivability. We developed both single-stage and two-stage MILP formulations for the weakly survivable routing problem. A two-stage solution approach for strongly survivable routing was also developed to determine a routing that minimizes spare capacity requirements before and after a physical link failure.

To mitigate the computational complexity of the MILP formulations developed, we developed heuristics for both weakly and strongly survivable routing problems and presented simulation results comparing the performance of these heuristics with respect to the performance of the MILP formulations. It was observed that the computation time of the heuristics is much less than that for the MILP approaches, while achieving a performance around 85% of the optimum.

While the formulations developed in this paper provide much insight into the problems considered, they also provide the basis for extensions such as design of logical topologies that permit survivable routing satisfying multiple cross-layer metrics: logical topology augmentation for guaranteed survivability, survivability against multiple physical link failures, maximizing MCLC, maximizing after-failure connectivity of the logical topology, maximizing logical capacity, and load balancing and energy-minimized survivable routing. Also, they provide the basis for developing approximation algorithms using sophisticated techniques such as relaxation methods, as well as the basis for estimating the approximation gap of the heuristic methods. These issues are under investigation.

ACKNOWLEDGMENTS

The research of Krishnaiyan Thulasiraman, Guoliang Xue, and Sartaj Sahni has been supported by National Science Foundation (NSF) grants 1115130, 1115129, and 1115184. The information reported here does not reflect the position or the policy of the federal government.

References

[1] A. Sano, T. Kobayashi, S. Yamanaka, A. Matsuura, H. Kawakami, T. Miyamoto, K. Ishihara, and H. Masuda, "102.3 Tb/s (224×548 Gb/s) C- and extended L-band all-Raman transmission over 240 km using PDM-64QAM single carrier FDM with digital pilot tone," in *Proc. Optical Fiber Communication Conf. (OFC/NFOEC)*, Los Angeles, CA, 2012, pp. 1–3.

[2] S. Ramamurthy, L. Sahasrabuddhe, and B. Mukherjee, "Survivable WDM mesh networks," *J. Lightwave Technol.*, vol. 21, no. 4, pp. 870–883, 2003.

[3] N. Ghani, S. Dixit, and T.-S. Wang, "On IP-over-WDM integration," *IEEE Commun. Mag.*, vol. 38, no. 3, pp. 72–84, 2000.

[4] T. Lin, Z. Zhou, and K. Thulasiraman, "Logical topology survivability in IP-over-WDM networks: Survivable lightpath routing for maximum logical topology capacity and minimum spare capacity requirements," in *Proc. Int. Workshop on Design of Reliable Communication Networks (DRCN)*, Krakow, 2011, pp. 1–8.

[5] R. M. Karp, *Reducibility Among Combinatorial Problems.* New York: Plenum, 1972, ch. 8, pp. 85–103.

[6] E. Modiano and A. Narula-Tam, "Survivable routing of logical topologies in WDM networks," in *Proc. IEEE INFOCOM*, Anchorage, AK, 2001, pp. 348–357.

[7] A. Todimala and B. Ramamurthy, "A scalable approach for survivable virtual topology routing in optical WDM networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 6, pp. 63–69, 2007.

[8] J. Strand, A. L. Chiu, and R. Tkach, "Issues for routing in the optical layer," *IEEE Commun. Mag.*, vol. 39, no. 2, pp. 81–87, 2001.

[9] K. Lee and E. Modiano, "Cross-layer survivability in WDM-based networks," in *Proc. IEEE INFOCOM*, April 2009, pp. 1017–1025.

[10] D. D.-J. Kan, A. Narula-Tam, and E. Modiano, "Lightpath routing and capacity assignment for survivable IP-over-WDM networks," in *Proc. Int. Workshop on Design of Reliable Communication Networks (DRCN)*, Washington, DC, 2009, pp. 37–44.

[11] M. Kurant and P. Thiran, "Survivable mapping algorithm by ring trimming (SMART) for large IP-over-WDM networks," in *Proc. Int. Conf. on Broadband Networks (BROADNETS)*, Oct. 2004, pp. 44–53.

[12] S. Lee, J.-C. Liu, and Y. Chen, "An ear-decomposition based approach for survivable routing in WDM networks," in *Proc. Int. Conf. on Advanced Information Networking and Applications (AINA)*, March 2005, pp. 459–464.

[13] M. S. Javed, K. Thulasiraman, M. A. Gaines, and G. Xue, "Survivability aware routing of logical topologies: On Thiran-Kurant approach, enhancements and evaluation," in *Proc. IEEE Global Communication Conf. (GLOBECOM)*, San Francisco, CA, 2006.

[14] M. Javed, K. Thulasiraman, and G. Xue, "Lightpaths routing for single link failure survivability in IP-over-WDM networks," *J. Opt. Commun. Netw.*, vol. 9, no. 4, pp. 394–401, 2007.

[15] K. Thulasiraman, M. S. Javed, and G. Xue, "Circuits/cutsets duality and a unified algorithmic framework for survivable logical topology design in IP-over-WDM optical networks," in *Proc. IEEE INFOCOM*, Rio de Janeiro, 2009, pp. 1026–1034.

[16] K. Thulasiraman, M. Javed, and G. Xue, "Primal meets dual: A generalized theory of logical topology survivability in IP-over-WDM optical networks," in *Proc. Int. Conf. on Communication Systems and Networks (COMSNETS)*, Bangalore, 2010, pp. 128–137.

[17] K. Thulasiraman, T. Lin, M. Javed, and G. Xue, "Logical topology augmentation for guaranteed survivability under multiple failures in IP-over-WDM optical networks," *Opt. Switching Networking*, vol. 7, no. 4, pp. 206–214, 2010.

[18] C. Liu and L. Ruan, "A new survivable mapping problem in IP-over-WDM networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 25–34, 2007.

[19] H. Kerivin, D. Nace, and T.-T.-L. Pham, "Design of capacitated survivable networks with a single facility," *IEEE/ACM Trans. Netw.*, vol. 13, no. 2, pp. 248–261, 2005.

[20] D. Rajan and A. Atamtürk, *Survivable Network Design: Routing of Flows and Slacks*, 1st ed. Kluwer Academic, 2003, pp. 65–81.

[21] D. Rajan and A. Atamtürk, "A directed cycle-based column-and-cut generation method for capacitated survivable network design," *Networks*, vol. 43, no. 4, pp. 201–211, 2004.

[22] Y. Liu, D. Tipper, and K. Vajanapoom, "Spare capacity allocation in two-layer networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 5, pp. 974–986, 2007.

[23] A. Haque and P. H. Ho, "A study on the design of survivable optical virtual private networks (o-VPN)," *IEEE Trans. Reliab.*, vol. 55, no. 3, pp. 516–524, 2006.

[24] C. Cavdar, A. Yayimli, and B. Mukherjee, "Multi-layer resilient design for layer-1 VPNs," in *Optical Fiber Communication Conf. and the Nat. Fiber Optic Engineers Conf. (OFC/NFOEC)*, San Diego, CA, 2008, pp. 1–3.

[25] Q. Deng, G. Sasaki, and C.-F. Su, "Survivable IP over WDM: an efficient mathematical programming problem formulation," in *Proc. 40th Allerton Conf. on Communication, Control, and Computing*, Monticello, IL, 2002.

[26] L. Sahasrabuddhe, S. Ramamurthy, and B. Mukherjee, "Fault management in IP-over-WDM networks: WDM protection versus IP restoration," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 1, pp. 21–33, 2002.

[27] "Library for efficient modeling and optimization in networks (LEMON)" [Online]. Available: http://lemon.cs.elte.hu/trac/lemon.

[28] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network Flows: Theory, Algorithms, and Applications*. Prentice-Hall, 1993.

[29] K. S. Ho and K. W. Cheung, "Generalized survivable network," *IEEE/ACM Trans. Netw.*, vol. 15, no. 4, pp. 750–760, 2007.

[30] "SNDlib" [Online]. Available: http://sndlib.zib.de/.

Tachun Lin (M'08) received his Ph.D. degree in computer science from the University of Oklahoma, Norman, OK, USA, in 2011 and his B.S. and M.S. degrees in computer science from National Chiao Tung University, Hsinchu, Taiwan, in 1999 and 2001, respectively. Since 2011, he has been an Assistant Professor of Computing and Technology at Cameron University, Lawton, OK. He was the Best Paper Award Runner-up of DRCN 2011 and RNDM 2012. His research interests include cross-layer network design, network optimization, and graph theory. He is a member of ACM, INFORMS, and IEEE.

Zhili Zhou (M'11) received her Ph.D. degree in industrial and system engineering from University of Florida, Gainesville, FL, USA, in 2010 and B.S. and M.S. degrees in mathematics from Nanjing University, Nanjing, China, in 2003 and 2006, respectively. She is a research staff member at IBM Research Collaboratory, Singapore. She took second place for the 2011 Pritsker Doctoral Dissertation

Award from the Institute of Industrial Engineers for her dissertation. She was the Best Paper Award Runner-up of DRCN 2011 and RNDM 2012. Her research interests are in mixed integer programming, combinatorial optimization, stochastic programming, and quality of service in communication networks, transportation networks, and power grids. She is a member of INFORMS and IEEE.

Krishnaiyan Thulasiraman (M'72-SM'84-F'90) received his bachelor's and master's degrees in electrical engineering from the University of Madras (1963, 1965) and the Ph.D. degree from the Indian Institute of Technology (IITM), Madras, India, in 1968. Since 1994 he has held the Hitachi Chair and has been a Professor with the School of Computer Science, University of Oklahoma, Norman. Prior to joining the University of Oklahoma, he was a Professor (1981–1994) in and Chair (1993–1994) of the Electrical and Computer Engineering Department, Concordia University, Montreal, Canada. He was on the faculty in the Electrical Engineering and Computer Science Departments of IITM from 1965 to 1981. He has published more than 100 papers in archival journals. He has coauthored, with M. N. S. Swamy, *Graphs, Networks, and Algorithms* (New York: Wiley, 1981), *Graphs: Theory and Algorithms* (New York: Wiley, 1992), and several book chapters. He has a US patent on distributed QoS routing. His research interests have been in graph theory, combinatorial optimization, algorithms, and applications in a variety of areas in computer science and electrical engineering, particularly fault diagnosis, tolerance, and testing. His most recent interest is in the emerging area of network science and engineering. He has received several awards and honors: Distinguished Alumnus Award of IITM, IEEE Circuits and Systems Society Technical Achievement Award, IEEE CAS Society Golden Jubilee Medal, Senior Fellow of the Japan Society for Promotion of Science, Gopalakrishnan Endowed Chair Professor of IITM, and Fellow of IEEE, AAAS, and the European Academy of Sciences.

Guoliang Xue (M'98-SM'99-F'11) received his B.S. degree in mathematics and M.S. degree in operations research from Qufu Normal University, Qufu, China, in 1981 and 1984, respectively, and his Ph.D. degree in computer science from the University of Minnesota, Minneapolis, MN, USA, in 1991. He is a Professor of Computer Science and Engineering with Arizona State University, Tempe, AZ, USA. His research interests include quality of service provisioning, resource allocation, survivability, security, and privacy issues in networks. He has received Best Paper Awards at IEEE Globecom 2007, IEEE ICC 2011, IEEE MASS 2011, IEEE Globecom 2011, and IEEE ICC 2012 and was a Best Paper Runner-up at IEEE ICNP 2010. He is an Associate Editor of the *IEEE/ACM Transactions on Networking* and *IEEE Network* and a member of the Standing Committee of IEEE INFOCOM. He is general co-chair of the 2014 IEEE Conference on Communications and Network Security (IEEE CNS 2014). He was a keynote speaker at IEEE LCN 2011, a TPC co-chair of IEEE INFOCOM 2010, and a co-organizer of the 2012 ARO Workshop on Human Centric Computing with Collective Intelligence—Challenges and Research Directions (HC3I 2012). He served on the editorial boards of *Computer Networks*, *IEEE Transactions on Wireless Communications*, and *IEEE Transactions on Circuits and Systems*. He was a distinguished lecturer of the IEEE Communications Society. He is an IEEE fellow.

Sartaj Sahni (M'78-SM'86–F'88) is a Distinguished Professor of Computer and Information Sciences and Engineering at the University of Florida. He is also a member of the European Academy of Sciences; a fellow of IEEE, ACM, AAAS, and the Minnesota Supercomputer Institute; and a distinguished alumnus of the Indian Institute of Technology, Kanpur. In 1997, he was awarded the IEEE Computer Society Taylor L. Booth Education Award "for contributions to Computer Science and Engineering education in the areas of data structures, algorithms, and parallel algorithms," and in 2003 he was awarded the IEEE Computer Society W. Wallace McDowell Award "for contributions to the theory of NP-hard and NP-complete problems." He was awarded the 2003 ACM Karl Karlstrom Outstanding Educator Award. He has published over 300 research papers and written 15 texts. His research publications are on the design and analysis of efficient algorithms, parallel computing, interconnection networks, design automation, and medical algorithms.